O K L A H O M A   S T A T E   U N I V E R S I T Y

S C H O O L   O F   E L E C T R I C A L   A N D   C O M P U T E R   E N G I N E E R I N G

ECEN 5713 Linear Systems
Spring 2000
Midterm Exam #2

Name : ______________________________

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Problem 1:
Extend the set

$$\begin{bmatrix} 1 \\ -1 \\ 2 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ -5 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -1 \\ 0 \\ -1 \end{bmatrix}$$

to form a basis in $(\Re^5, \Re)$.

Problem 2:
For

$$A = \begin{bmatrix} 1 & -1 & 3 \\ 2 & 5 & 1 \\ -1 & 0 & 2 \\ 3 & 7 & 1 \end{bmatrix},$$

determine the rank and nullity of the above linear operator, A ? And find a basis for the range space and the null space of the linear operator, A, respectively ?

Problem 3:

Show if the following sets

$$\begin{bmatrix} 3 \\ 1 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \\ -1 \end{bmatrix} \text{ and } \begin{bmatrix} 2 \\ 0 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ -2 \\ 2 \end{bmatrix}$$

span the same subspace V of $(\Re^4, \Re)$.

<u>Problem 4</u>:

An orthogonal complement space, $V^{\perp}$, is spanned by v_1, v_2, v_3 given as

$$v_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad v_3 = \begin{bmatrix} -1 \\ 2 \\ 2 \\ 1 \end{bmatrix}.$$

Determine the original space, *V*, and find an orthogonal basis. For $x = \begin{bmatrix} 0 & 3 & 3 & 0 \end{bmatrix}^T$, find its direct sum representation of $x = x_1 \oplus x_2$, such that $x_1 \in V$, and $x_2 \in V^{\perp}$.